

STATISTICS OF COSMOLOGICAL INHOMOGENEITIES

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ABSTRACT

This contribution to the Proceedings is based on the talk given at the Conference on Birth of the Universe and Fundamental Physics, Rome, May 18-21, 1994. Some selected topics of the subject are reviewed: Models of Primordial Fluctuations; Reconstruction of the Cosmological Density Probability Distribution Function (PDF) from Cumulants; PDFs in the Zel'dovich Approximation and from Summarizing Perturbation Series; Fitting by the Log-normal Distribution.

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1 Introduction

The canonical model for the formation of large-scale structure is based on gravitational instability of small initial fluctuations in the matter density δ . The primordial fluctuations are assumed to be a random field, which is fully specified by the joint probability density functions (hereafter PDFs). These are assumed to originate from quantum fluctuations of the inflaton scalar field $\delta\phi_k e^{ikx}$ that were stretched to large comoving scales during the inflation phase [1]. For the simplest case of a single inflaton scalar field, curvature (adiabatic) density perturbations are generated by the inflaton fluctuations, $\delta \propto \delta\phi$, which is a superposition of spatial waves e^{ikx} with random phases, realizes a random Gaussian field.

Theoretically, there are scenarios based on non-Gaussian initial fluctuations. Although these models currently do not appear to be our first choice, they are interesting as alternatives. I will discuss these models in the next Section. The statistical nature of the initial fluctuations is therefore a basic distinguishing feature between competing theories.

The COBE DMR temperature anisotropy fluctuations sky map, in principle, can directly probe the statistics of the primordial fluctuations on large scales. It is consistent with Gaussian fluctuations, but its signal to noise level so far is sufficient only to rule out strongly non Gaussian models [2].

In any case, at present the smoothed galaxy distribution is not a Gaussian random field, due to the nonlinear dynamics of gravitational instability. During linear evolution, when all Fourier components δ_k evolve at the same rate, the cosmic PDFs do not change form. However, even quasilinear evolution, which makes phases correlated, introduces strong non-Gaussian features.

The study of the PDFs of the large scale cosmic density and velocity fields, and their moments drew much attention recently. In Section 3, I will discuss recent analytic results on the derivation of the density and velocity PDFs from gravitational dynamics. There were suggestions to design the density PDF phenomenologically [3, 4]. For instance, the log-normal form is a surprisingly good fit for the density PDF in CDM model. Why it may be so, I will discuss in Section 4.

The discrete analogy of the one point PDF is the counts in cell statistics. The density PDF is obtained by the smoothing of the discrete galaxy field with a sharp filter. The observed density PDF manifests significant non-Gaussian features in the non-linear and even in the quasi-linear regimes, and can be well fitted by the log-normal distribution. Assuming that galaxies trace mass, observed galaxy distribution is consistent with Gaussian initial fluctuations on scales of these surveys. However, the discriminatory power of the observed PDFs is currently limited for several reasons. I will discuss these issues in the Conclusion.

2 Models of Primordial Fluctuations

It has been shown that in the framework of the the inflation picture there is still room for non-Gaussian fluctuations. In case of the curvature perturbations it requires specific forms of the inflaton potential or coupling with other fields, plus fine tuning of the parameters [6]. On the contrary, in case of the isocurvature perturbations generation of non Gaussian fluctuations is rather typical [6, 7]. Let another light scalar field χ be present at the inflation. Long wavelengths fluctuations $\delta\chi$ inevitably arise during inflation. If later the χ -particles become dominant and responsible for dark matter, then we come to the model with isocurvature perturbations. The fluctuations $\delta\chi$ are Gaussian. However, they can generate the isocurvature energy density perturbations $\delta_{isocurv} = F(\delta\chi)$, which are non Gaussian for nonlinear functions F (local non Gaussian field [5]). An interesting example is the cosmic axion as χ -field. The axion energy density is proportional to the axion potential $V(\chi) = V_0(1 - \cos \chi/\chi_0)$, therefore isocurvature fluctuations in the model with axions as CDM are non Gaussian. The power spectrum of isocurvature fluctuations does not depend on their statistics. Unfortunately, the observed power spectrum ranging from the horizon to the galaxy clustering scales leaves small room for the CDM scenario with mixture of curvature and isocurvature fluctuations.

Another possibility is that the χ -particles are underdominant but their fluctuations are significant and play the role of the seeds for the structure formation [7]. There are models of the local non Gaussian isocurvature baryon fluctuations [8]. Non-Gaussian density fluctuations also arise in scenarios, where they originate from topological defects, such as cosmic strings [9] or textures [13], or late phase transition [11] or from non-gravitational cosmic explosions [12]. Initial shape $P(\delta)$ in scenarios with topological defects contains the long tail in the overdense region, for instance, exponential for textures. It is interesting that the velocity PDF in these model is normal, due to the central limit theorem [13].

3 Reconstruction of PDFs from Cumulants

First, let us recall basic terms which are adopted in the literature. We use the cumulants of the distribution $\langle \delta^p \rangle_c$ rather than its moments $\langle \delta^p \rangle$. The generating function of the cumulants is $C(\mu) = \sum_{p=2}^{\infty} \langle \delta^p \rangle_c \mu^p / p!$, where μ is an auxiliary parameter. All cumulants of the normal distribution are vanishing, except the linear variance of the density fluctuations σ . It is convenient to use rescaled cumulants $S_p(\sigma) = \langle \delta^p \rangle_c / \sigma^{2(p-1)}$, as a set of descriptive constants of a distribution. The S_p parameters would be constants in the hierarchical ansatz, but in general they are functions of σ . The parameter S_3 multiplied by σ is the skewness, and S_4 multiplied by σ^2 is the kurtosis. The advantage to use these parameters is that they are final numbers $S_p(0)$ for an arbitrarily small σ . The parameters $S_p(0)$ are the fingerprints of the particular dynamical model.

When the whole series of the cumulants is known it is possible to reconstruct the density PDF

from the generating function of the cumulants

$$P(\rho) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} d\mu \exp[-C(\mu) - (\rho - 1)\mu]. \quad (1)$$

Here and further ρ is the density in units of mean density. In practice we may have only a few lower cumulants, such as the skewness and the kurtosis. Theoretically, they can be calculated in the perturbation theory. In the case of the weakly non-linear dynamics, when a slight departure from the initial Gaussian distribution is expected, one can obtain the general decomposition series around the Gaussian PDF induced by the first non-zero cumulants – the Edgeworth expansion. In cosmological context this was implicated in [14, 15, 16].

The Edgeworth expansion for the cosmic density PDF can be obtained from the reconstruction formula (1), keeping a few lowest terms in the generating function $C(\mu)$. The result is

$$P(\delta) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{\delta^2}{2\sigma^2}} \left[1 + \sigma \frac{S_3}{6} H_3(\nu) + \sigma^2 \left(\frac{S_4}{24} H_4(\nu) + \frac{S_3^2}{72} H_6(\nu) \right) + \dots \right] \quad (2)$$

where $H_n(\nu)$ are the Hermite polynomials, $\nu = \delta/\sigma$. The actual forms of the parameters $S_p = S_p(\sigma)$ which depend on the particular dynamics, affect the expansion (2) with respect to σ . The usual measurements of the lowest cumulants are significantly affected by the high density tail of the PDF, i.e. the rare events. It is interesting, that the lowest cumulants alone are responsible for the shift of the peak of $P(\delta)$. Therefore the measurement of the shape of the PDF maximum, which statistically is more robust, can provide an alternative method of evaluation of the lowest cumulants. The Edgeworth expansion fails to reproduce $P(\delta)$ when $|\delta| \geq 0.5$ where spurious wiggles appear. In practice it is useless for $\sigma \geq 0.3$. The Edgeworth expansion also can be used for modelling slightly non Gaussian initial fluctuations.

4 PDFs in the Zel'dovich Approximation

Physical meaning of the density PDF is the fraction of volume with a given level of density. The former shape of the initial density PDF is broken as non-linearity develops, since matter is evacuated from the underdense regions, which occupy the largest fraction of volume, meanwhile the overdense regions collapse. In the formal expression (2) this corresponds to the positive sign of the skewness, increasing with time. This feature of the density field evolution is clearly manifested in the Zel'dovich approximation

$$\rho = \frac{\rho_0}{|(1 - D\lambda_{01})(1 - D\lambda_{02})(1 - D\lambda_{03})|}, \quad (3)$$

where ρ_0 is the initial local density, growing mode of adiabatic perturbations is $D(t)$, and λ_{0i} are the eigenvalues of the Lagrangian deformation tensor. This formula does not assume any particular initial statistics, whether Gaussian or not. The gravitational clustering at sufficiently large scales can be considered in the quasilinear theory in a single stream regime ignoring small scale details.

The Zel'dovich approximation (3) can be applied for the filtered initial gravitational potential. This approach sometimes is referred to as the truncated Zel'dovich approximation.

In the truncated Zel'dovich approximation the statistics of the evolved density field can then entirely be obtained from the statistics of the initial local density ρ_0 and the initial eigenvalues λ_0 -s. For adiabatical perturbations ρ_0 is the background mean density. Let the initial joint PDF of all involved cosmic fields, including velocity and gravitational potential, be $W_0(\rho_0, \lambda_{01}, \lambda_{02}, \lambda_{03}, \vec{u}_0, \Phi_0)$.

The density PDF can be obtained in the general case of arbitrary initial statistics simply as an integral over all involved variables except density [15, 16]

$$P(\rho) = \int \delta[\rho|(1 - D\lambda_{01})(1 - D\lambda_{02})(1 - D\lambda_{03})| - 1] W_0 d\lambda_{01} d\lambda_{02} d\lambda_{03} d^3 u_0 d\Phi_0, \quad (4)$$

For the Gaussian initial conditions we can omit \vec{u}_0 and Φ_0 in the initial joint PDF and write it as $W_0(\rho_0, \lambda_{01}, \lambda_{02}, \lambda_{03}) = \delta(\rho_0 - 1) M_0(\lambda_{01}, \lambda_{02}, \lambda_{03})$. The second factor is the well known joint distribution function of the eigenvalues of the initial deformation tensor for an initial Gaussian displacement field [17]. Substituting this form of W_0 into the integral (4), after some tedious algebra, we can reduce it to the one-dimensional integral [18, 19]:

$$P(\rho) = \frac{N}{\rho^3} \int_{3(\frac{\rho}{\rho_0})^{1/3}}^{\infty} ds e^{-(s-3)^2/2\sigma^2} (1 + e^{-6s/\sigma^2}) (e^{-\beta_1^2/2\sigma^2} + e^{-\beta_2^2/2\sigma^2} - e^{-\beta_3^2/2\sigma^2}) \quad (5)$$

where $\beta_n(s) \equiv s \cdot 5^{1/2} \left(\frac{1}{2} + \cos \left[\frac{2}{3}(n-1)\pi + \frac{1}{3} \arccos \left(\frac{54\rho^3}{\rho_0^3} - 1 \right) \right] \right)$, and prefactor is $N = \frac{9 \cdot 5^{3/2}}{4\pi\sigma^4}$. In the limit of small σ the expression (5) transfers into the form (2). The density PDF calculated numerically from formula (5) is plotted in Fig. 1. Without final smoothing, this PDF does not depend on the power spectrum n . The quality of approximation of actual PDF by the formula (5) is increasing as n is decreasing $n \rightarrow -3$. The n -dependence can be taken into account in the limit of small σ [20].

For the slightly non Gaussian initial fluctuations, one can expand the joint distribution $W_0(\rho_0, \lambda_{01}, \lambda_{02}, \lambda_{03}, \vec{u}_0, \Phi_0)$ around its Gaussian form, discussed above. For this purpose it is convenient to return to six components of the deformation tensor, and apply the generalized Edgeworth expansion for several variables. From (4) one can obtain the density PDF evolving in time from slightly non Gaussian fluctuations in form of series around distribution (5). One of the lesson from this exercise is that, in principle, the final density statistics depends on the statistics of all fields involved in the joint distribution $W_0(\rho_0, \lambda_{01}, \lambda_{02}, \lambda_{03}, \vec{u}_0, \Phi_0)$.

5 PDFs from Summarizing Perturbation Series

Using the perturbation theory for Gaussian initial conditions, it is possible to derive the cumulants of the density PDF in the small σ limit. The basic assumption here is that the gravitational clustering at sufficiently large scales can be considered in the single stream regime. Derivation

of the lowest cumulants in the quasi-linear dynamics in the single stream regime was extensively elaborated [20, 21, 22, 23, 24]. Bernardeau [25, 26] found an elegant method to derive the closed form for the generating function of the cumulants $C(\mu)$ in the limit of small σ , which allows to obtain all parameters $S_p(0)$ and reconstruct the density PDF in this regime.

The chain of equations for density perturbation $\delta^{(p)}$ in each order p cannot be resolved. However, one can define the connected averages: $\nu_p \equiv \langle \delta^{(p)} (\delta^{(1)})^p \rangle_c / \sigma^{2p}$, and construct their generating function $G(\tau) = \sum_{p=1}^{\infty} (-\tau)^p \nu_p / p!$, where τ is an auxiliary variable. It is remarkable that there is a single equation for $G(\tau)$ which corresponds to the density contrast of the spherical collapsing linear overdensity τ [29]. An approximated analytical solution for this function is

$$G(\tau) = \frac{1}{(1 + \tau/\alpha)^\alpha} - 1. \quad (6)$$

For the three dimensional single stream cosmological gravitational instability $\alpha \approx 1.5$ [26].

This method and results are rather general in the limit of small σ . For instance [16], for the Zel'dovich dynamics in the one dimensional case, where the Zel'dovich ansatz is an exact solution, the formula (6) is valid with $\alpha = 1$. For the Zel'dovich approximation in the two and three dimensional cases, $\alpha = N$, where N is the space dimension. In the formal limit $N \gg 1$ we get $G(\tau) = \exp(-\tau) - 1$, which coincides with that of the log-normal distribution.

The transition from $G(\tau)$ to the generating function of the cumulants $C(\mu)$ is given by the Legendre transform with the variable of the transform equal to unity afterwards [32, 33]. Substituting the solution (6) into (1), one reconstructs the density PDF. Without final smoothing, this PDF depends on σ only and not on the power spectrum index n . However, for a practical purpose it is necessary to filter the evolved density field. After filtering, density cumulants and PDF depend on the shape of the power spectrum [22, 24]. Bernardeau demonstrated [27], that the effect of filtering reduces to the a simple transformation of the generating function $G(\tau) \rightarrow G^f(\tau)$. The resulting shape of the density PDF, based on the filtered generating function $G^f(\tau)$, is presented in Fig. 1, 2.

6 Fitting By The Log-Normal Distribution

As it was noted a long time ago by Hubble [30], the galaxy count distribution in angular cells on the sky might be well described by a log-normal distribution. The recent study of the spatial distribution of galaxy count are in good agreement with the log-normal fit [31, 32, 19] The log-normal density distribution reads

$$P(\rho) = \frac{1}{(2\pi\sigma_l^2)^{1/2}} \exp \left[-\frac{(\ln \rho + \sigma_l^2/2)^2}{2\sigma_l^2} \right] \frac{1}{\rho}, \quad \sigma_l^2 = \ln(1 + \sigma^2). \quad (7)$$

It was found [23, 20] that the log-normal distribution is an excellent approximation to the density PDF from N-body CDM simulation for the tested values of $0.3 < \sigma < 1.5$ in the tested range of

$\rho \leq 10$, see Fig. 1. Such a remarkable fitting inspires the thought that there might be a dynamical reason to manifest the log-normal features of the density PDF. In [3] the log-normal mapping of the linear density field was suggested to describe its non-linear evolution. This log-normal model is universal for any spectral index n . Unfortunately the log-normal mapping does not work [33]. At Fig. 2 the log-normal distribution is compared with the analytically derived density PDF [34]. The log-normal shape fits an actual PDF not for arbitrary n , but for $n \approx -1$. It is easy to understand, considering the cumulants of the log-normal distribution. For instance, we have $S_3^{log}(\sigma) = 3 + \sigma^2$ for an arbitrary σ .

The actual parameter is $S_3(0) = \frac{34}{7} - (n + 3)$. The parameter of the log-normal distribution $S_3^{log}(\sigma)$ fits an actual parameter not for arbitrary n and σ , but along the curve $n = -1.14 - \sigma^2$. The σ -correction to the $S_3(0)$ changes slightly the numerical prefactor of σ . The parameter $S_4^{log}(\sigma)$ fits an actual parameter $S_4(\sigma)$ approximately along the same curve. Thus, the log-normal distribution fits well in the particular region of the parameter space (n, σ) around the “log-normal” curve $n(\sigma)$, but worsening outside of this region. The actual density PDF can be further approximated by the Edgeworth expansion around the log-normal form [35]. By chance, the popular CDM model at moderate σ crosses this region.

Equipped with this method, one can understand why and where the other fits such as “thermodinamic” [4, 36] or the negative binomial distribution [32, 37] are applicable.

7 Discussion

In general case, S_p -parameters as functions of the linear density variance can be presented as series

$$S_p(\sigma, n) = S_p(0, n) + T_p(n)\sigma^2 + \dots \quad (8)$$

There is derivation of all coefficients $S_p(0, n)$ [25], but a little is known about $T_p(n)$, which describe a lowest σ -correction. Apparently, $T_p(n)$ depends on n . Some N-body simulations indicate the tendency $S_p(\sigma, n)$ to increase with σ faster as n decreases [38]. Parameters S_p slowly increase with σ at quasilinear regime, $T_p(n) \ll S_p(0)$, at least for $-1 \leq n \leq 0$ [14] and for CDM model [16]. A possible explanation is that the spherical collapse is the dominated form for these spectra, contrary to the pancakes for $n \rightarrow -3$. Another choice of the set of descriptive parameters is $\langle \delta^p \rangle / \sigma_{nl}^{2(p-1)}$, where nonlinear variance is $\sigma_{nl}(\sigma)$. It was shown in [41] numerically for CDM model that these combinations remain constants equal to $S_p(0)$ for a wide range of σ_{nl} . Therefore Bernardeau’s method, formally applicable for small σ , can be extrapolated across the whole quasilinear regime, which makes it very useful for practical application. The σ -dependence in S_p can be more significant for $n \rightarrow -3$. In this case the accuracy of PDF based on the Zel’dovich approximation, is increasing. Another related question is the hierarchical ansatz in the highly nonlinear regime. It assumes that $S_p(\sigma)$ are saturated as $\sigma \gg 1$. This is still an unanswered question, however, there are interesting results in this regime [39, 40].

In practice, galaxy count in cell distributions, and consequently, the density PDFs have been

measured in various catalogs in many works, for example see [31, 37, 42] for the CfA survey, [32, 19] the IRAS surveys, [42] for SSRS survey, in 43 for clusters of galaxies, in [44] for S_p parameters from the APM survey, and references therein. These data are compatible with Gaussian initial fluctuations, and can be fitted by the log-normal distribution, see, e.g., Fig. 3. Thus, some “log-normal” features of the observed density PDF would mean that the realistic cosmological model has the $n(\sigma)$ dependence close to the CDM-type models. Unfortunately, the error bars of the observed density PDF increase at large $\rho \sim 2 - 3$, and significantly affected by the depth of the sample [32, 23]. Therefore currently one can rule out only strongly non Gaussian models. Another potential problem is galaxy bias, which affects the observed deviation from Gaussianity [45]. The linear bias at filtering scales would preserve the reported analytic results. However, the departure from this simple rule is expected, as predicted by numerical simulations [46].

The density PDF deviates from a normal distribution very rapidly. On the contrary, the velocity PDF departures from its initial distribution very slowly [19, 47]. The observed velocity PDF is shown at Fig. 3.

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Fig. 1 The density PDF obtained by different methods for CDM initial conditions [20]. The points are measured in a numerical simulation at the time corresponding to the present epoch, at two different smoothing radii $R_0 = 5h^{-1}\text{Mpc}$ (triangles) and $R_0 = 15h^{-1}\text{Mpc}$ (circles). The rms density fluctuation are then respectively $\sigma = 1.52$ and $\sigma = 0.47$. The error bars have been obtained by dividing the sample into eight subsamples. The solid line is the prediction of the Bernardeau's method when the smoothing effects are taken into account. The dashed line is the prediction (5) from the ZA and the long dashed line is the lognormal distribution (7).

Fig. 2 The dynamically motivated density PDF for different n , $\sigma = 0.5$, versus log-normal distribution [34]. The solid line for $n = +1, 0, -1$ is the density PDF obtained by the Bernardeau method with the final smoothing. The solid line for $n = -2$ is the PDF in the Zel'dovich approximation. The dashed line is the log-normal distribution.

Fig. 3 PDFs for IRAS 1.9Jy density and velocity fields in a sphere of radius $80h^{-1}\text{Mpc}$, after Gaussian filtering with $R_s = 6h^{-1}\text{Mpc}$. Dashed and long dashed curves are the Gaussian and lognormal distributions with the same σ . Also shown the errors associated with the limited volume sampled [19].